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Sovereign Default Risk in a Monetary Union

Betty C. Daniel* and Christos Shiamptanis**

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Abstract

A country entering a monetary union gives up the right to determine its own monetary policy, thereby relinquishing monetary instruments to assure fiscal solvency. In this paper, we develop a new theoretical model to address fiscal solvency risk. We show that when debt is subject to an upper bound and policy faces stochastic shocks, a government can find itself in a position for which the expected present value of future surpluses under current policy is less than debt. Agents refuse to lend into such a position, and the sudden stop of capital flows defines a fiscal solvency crisis. We model the dynamics of a fiscal solvency crisis in a monetary union under the assumption that the fiscal authority will respond to the crisis using default to reduce the value of debt. We simulate the model to estimate fiscal solvency risk in the European Monetary Union. We find that countries adhering to the Stability and Growth Pack limits are perfectly safe, while countries like Greece and Italy, whose debt relative to GDP has strayed far above the 60 percent limit, are not.

Keywords: European Monetary Union, sovereign default, financial crisis.
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1 Introduction

A government, whose debt is denominated in its own currency, need never face default. In the event of insolvency, created by an unexpected spending need or revenue shortfall, the country can always restore solvency with seigniorage (Sargent and Wallace 1981) and/or debt devaluation through unexpected inflation.[1] Monetary union eliminates these instruments unless the union is willing to sacrifice price stability to restore solvency for an individual country. The European Central Bank has a single mandate of price stability, and no explicit mandate for individual country fiscal solvency.

Founders of the European Monetary Union were concerned that fiscal insolvency could threaten the currency union. They sought to replace the loss of individual-country monetary policy instruments with more prudent fiscal policy by placing limits on debt and the deficit. However, the world-wide financial crisis and recession, which began in 2007 and accelerated in 2008, has had profoundly negative consequences for government budget deficits and debt, with almost all countries in violation of the limits. In 2009 and 2010, interest rates for some countries relative to German rates spiked, reflecting market concern that these countries might default on their debt. What does economic theory have to offer about market concerns that some EMU countries could default?

In traditional models of sovereign default, a sovereign defaults when benefits to default exceed costs, typically modeled either as exclusion from future capital markets or as

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[1] The government can allow real returns on nominal government debt to be state-contingent even though nominal returns, as measured by nominal interest rates, are not (Chari, Christiano, and Kehoe 1991). This is achieved by surprise changes in the price level, which affect the real value of government debt, and is the mechanism in the "Fiscal Theory of the Price Level" (FTPL) (Sims 1994, Woodford 1994).

sanctions on output.[2] Since these punishments are independent of the magnitude of default in the traditional models, the magnitude is always one hundred percent.[3] Also, the cost of these punishments, compared with the benefits of complete debt repudiation, are not large enough to support magnitudes of debt empirically observed.[4] Additionally, the traditional models ignore the distortionary effect of taxation with the assumption that the central-planner-sovereign can seize any amount of GDP to pay debt obligations, and, hence, is always solvent. The deadly riots in the streets of Athens in May 2010, in response to announced austerity measures, are a painful contradiction to this assumption. Even in less extreme situations, a sovereign is limited in the amount of tax revenue, relative to GDP, that it can raise. Even if a sovereign, weighing costs and benefits, wanted to repay debt, there are limits on what it can repay. Perhaps the market is trying to decide if Greece has the ability to honor its debt, a solvency concern, not addressed in the traditional literature on sovereign default.

Financial fragility models offer an alternative explanation of sovereign default as one of several multiple equilibria once fundamentals become weak (Cole and Kehoe, 1996, 2000). The implication of these models is that, among the countries with weak fundamentals, actual default will be determined by exogenous self-fulfilling expectations of default, where default is again one hundred percent. In actual experience, default is never complete.

In this paper, we argue that interest rate spikes in Greece and several other EMU

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[2] The seminal paper using exclusion from future capital markets is Eaton and Gersovitz (1981). Obstfeld and Rogoff (1996) discuss sanctions. Eaton and Fernandez (1995) provide a survey.

[3] Yue (2010) modifies the traditional model by allowing default magnitude of less than one hundred percent to be determined by bargaining.

[4] Greek debt/GDP was 114.88% at the end of 2009, and the OECD forecasts it to rise to 130.2% by the end of 2011. Arellano and Heathcote (2009) find that credit market exclusion can sustain debt ceilings of between one and three percent of GDP. Yue (2010) allows for partial debt recovery and obtains higher values for debt ceilings around ten percent of GDP.

countries, relative to Germany, reflect a potential fiscal solvency crisis, not a discretionary sovereign default crisis or a bad outcome of multiple equilibria. We develop a new model of sovereign default as the policy response to a fiscal solvency crises. Bohn (1998, 2007) has shown that a positive response of a government's primary surplus to debt is sufficient to assure intertemporal government budget balance, implying fiscal sustainability and solvency. However, every government faces limits on its ability to raise taxes and reduce spending, which implies an upper bound on the present value of the primary surplus relative to GDP and an upper bound on the value of debt relative to GDP which the primary surplus can service.[5] Fiscal policy, relating the government primary surplus to outstanding debt, can be combined with the government's flow budget constraint to yield an expected time path for debt. When there is an upper bound on debt, current fiscal policy is sustainable only when debt does not violate the upper bound along this expected time path.

Therefore, our requirement for fiscal sustainability is stronger than Bohn's and has two parts. First, the fiscal response to debt must be strong enough that debt is expected to reach a long-run equilibrium which does not exceed its upper bound. Bohn's criteria allows debt to grow as long as it grows more slowly than the growth-adjusted interest rate. Second, in the transition to the long-run, debt cannot reach levels greater than the maximum present value of future surpluses. This is because the surpluses necessary to service that debt would be infeasible. The upper bound together with stochastic fiscal shocks implies that a fiscal policy, classified as sustainable according to criteria in Bohn

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[5] Other authors are beginning to model the implications of upper bounds on debt, including Bi (2010), Bi and Leeper (2010), and Davig, Leeper, and Walker (2010).

(1998, 2007), could violate the upper bound in the long run or have a positive probability of violating the upper bound in the transition to the long run, and therefore be unsustainable according to the stronger criteria.

The first contribution of this paper is the development of a new theoretical model to address fiscal solvency risk in the presence of an upper bound on debt and stochastic shocks to the government's primary surplus. If fiscal shocks, together with expectations of debt devaluation, send debt onto a path which violates the upper bound, agents refuse to lend because they cannot expect market rates of return. The sudden stop in lending defines the fiscal solvency crisis. The dynamics in the neighborhood of a fiscal solvency crisis are partially determined by expectations of the policy response to restore equilibrium. We focus on a policy response in which the crisis country reduces the magnitude of debt through partial default to restore fiscal solvency.[6] The default response allows us to present a new model of sovereign default as a consequence of fiscal insolvency, in contrast to the traditional sovereign default models.[7] We show that default without fiscal reform leaves markets turbulent with additional defaults following the initial one. We also briefly consider other policy responses, including a decision by the central bank to restore solvency with a change in monetary policy for the union, fiscal reform raising the present value of future surpluses, and fiscal transfers from solvent member countries.

Our second contribution is that we simulate the model to provide estimates of fiscal solvency risk in the EMU. We find that the probability of a fiscal solvency crisis over

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[6] The model extends the model of fiscal risk introduced by Daniel (2010), by allowing for a more general fiscal policy and by focusing on the policy response of default.

[7] Bi and Leeper (2010) present a sovereign default model in which the sovereign defaults if its stochastic upper bound on debt, modeled as the largest debt the government is willing to service, exceeds actual debt.

a horizon of ten years is zero for countries with values of debt and the primary surplus bound by the Maastricht Treaty limits. For countries like Greece and Italy, in which 2009 debt strayed far above these limits, crisis probability over the next ten years, based on 2009 debt and surpluses is positive, but small.[8] However, small changes in baseline parameters, or increases in debt to levels projected by the OECD for 2011 can increase the probability of a crisis substantially. Once expectations of a default become positive, the probability of a crisis rises at an increasing rate due to the effect of expectations of default on interest rates and, therefore, on the rate of debt accumulation. Other papers provide estimates of fiscal risk, based on VAR models of debt, but this risk is that of debt relative to GDP reaching an upper bound (Garcia and Rigobon 2004), or beginning to grow (Tanner and Samake 2008), over a particular horizon. Neither of these events need cause a crisis, and both measures miss the endogenous acceleration of the growth of debt in the neighborhood of a crisis due to the effect of expectations on interest rates.

This paper is organized as follows. The second section describes the new theoretical model. The third section considers dynamics leading to a fiscal crisis. The fourth section contains simulations of fiscal risk. The fifth section concludes.

2 Model

2.1 Overview

In this section, we set up a simple model of a monetary union which we can use to address fiscal solvency risk. The model contains four key assumptions. First, international creditors lend to a government only when they expect to receive the market rate of return.

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[8] These numbers have been revised since we conducted our simulations.

Second, there is an upper bound on the present value of primary surpluses relative to output which a country can sustain. Third, fiscal policy is subject to stochastic shocks. Together the upper bound and stochastic shocks imply risk on government debt, reflecting the concern by the EMU founders and the reality that a government's commitment to raise taxes cannot be totally unconditional. Fourth, institutions are strong enough that a solvent government always repays.

We fill out the model with enough structure to obtain an equation for the evolution of government debt relative to output. This requires specification of monetary and fiscal policy as well as government budget constraints. We assume that the monetary authority has a price level target and that the fiscal authority follows a rule relating the current primary surplus to past debt. The rule is subject to stochastic shocks, giving fiscal policy risk. The rule we choose is simple and does not require full specification of a general equilibrium model. However, any rule with fiscal risk could be used to complete the model.

2.2 Goods and Asset Markets

We assume that the monetary union consists of N countries. The $j = 1, 2, ..., N$ countries are small enough that they cannot affect the world price level or world interest rate. There is a single good in the world, implying that equilibrium in goods markets requires the law of one price. Normalizing the world price level at unity and assuming no world inflation implies that the equilibrium price level in the monetary union is the exchange rate.

The **first key assumption** is that international creditors are willing to buy and sell country j's government bonds as long as its interest rate, i_{jt}, satisfies interest rate parity.

Interest rate parity can be derived as the Euler equation for a representative world agent when the covariance of the country j interest rate with world-agent consumption is zero, or when the world agent is risk neutral. Under the additional assumptions that the world interest rate (i) is constant, interest rate parity can be expressed as

$$\frac{1}{1+i_{jt}} = \left(\frac{1}{1+i}\right) E_t \left[\frac{P_t}{P_{t+1}} \delta_{jt+1}\right], \qquad j = 1, 2, ...N \tag{1}$$

where E_t denotes the expectation conditional on time t information, P_t denotes the price level in the monetary union, and δ_{jt+1} is the fraction of the value of the j country's bond that will be repaid in period $t+1$.

Interest rates in the monetary union countries can rise above the world interest rate when there is some possibility of a crisis which will be resolved with either default ($\delta_{jt+1} < 1$) or inflation $\left(\frac{P_t}{P_{t+1}} < 1\right)$. If default is used to resolve fiscal crises, then a country with a positive probability of default in the next period, such that $E_t \delta_{jt+1} < 1$, would have an interest rate which is higher than the rate in other member countries for which there is no probability of default. If default is ruled out as a policy response to a crisis, then $\delta_{jt+1} \equiv 1$ $\forall j$, t, and all N member-country interest rates are equal. They can be higher than the world rate when there is some probability that debt devaluation through unexpected inflation will be used to resolve a crisis. In this paper, we focus on the default response and only briefly consider unexpected inflation.[9]

2.3 Monetary Policy

Monetary policy is assumed to have a fixed price level target, implying an inflation target of zero. When there is no probability of debt devaluation through either default or

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[9] See Daniel (2010) and Daniel and Shiamptanis (2010) for a more complete analysis of a policy-response allowing surprise inflation.

inflation in the next period for any of the N countries, the price level is fixed at its target and interest rate parity from equation (1) implies that the nominal interest rates for all countries are equal at $i_{jt} = i$.[10] When there is a positive probability of debt devaluation, we assume that the monetary authority maintains its price-level target and lets the interest rate rise to satisfy equation (1).

2.4 Fiscal Policy

2.4.1 Government Flow Budget Constraint

We assume that member governments issue bonds denominated in the common currency. Assuming that a fraction, η_j, of the union's money supply, M_t, is supported by country $j's$ domestic bonds, a member country's nominal flow government budget constraint is given by

$$B_{jt} + \eta_j M_t = \delta_{jt} \left[(1 + i_{t-1}) B_{jt-1} + \eta_j M_{t-1} \right] + G_{jt} - \tau_{jt} P_t Y_{jt},$$

where B_{jt} is nominal government bonds held by the public, G_{jt} is nominal government expenditures, Y_{jt} is real output and τ_{jt} is the tax rate on nominal output of country j. Letting small letters denote values relative to output and dropping the j notation to simplify, the values of debt relative to output and the primary surplus relative to output for country j can be expressed respectively as

$$b_t = \frac{1}{P_t Y_t} \left(B_t + \frac{1}{1 + i_t} \eta M_t \right),$$

$$s_t = \frac{1}{P_t Y_t} \left(\tau_t P_t Y_t - G_t + \left(\frac{i_t}{1 + i_t} \right) \eta M_t \right).$$

Allowing for inflation and default, either of which could be created by a fiscal solvency crisis, the government's flow budget constraint can be expressed in terms of debt and the

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[10] This policy could be implemented with a Taylor rule, whereby the monetary authority follows the Taylor principle, raising the nominal interest rate by more than any price level increase.

primary surplus relative to output as[11]

$$b_t = \left(\frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right)b_{t-1} - s_t, \tag{2}$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation rate, and $g = \frac{Y_t}{Y_{t-1}} - 1$ is the real output growth rate.[12] Imposing interest rate parity from equation (1) and defining γ_t as capital loss on the outstanding stock of debt, such that

$$\gamma_t = \left(1 - \frac{\delta_t}{1+\pi_t}\right)\left(\frac{1+i_{t-1}}{1+g}\right)b_{t-1},$$

the equation for the evolution of debt relative to output can be expressed as

$$b_t = (1+r)\,b_{t-1} - s_t - (\gamma_t - E_{t-1}\gamma_t)\,. \tag{3}$$

The growth-adjusted interest rate is denoted by $r = \left(\frac{i-g}{1+g}\right)$, and $(\gamma_t - E_{t-1}\gamma_t)$ is the unexpected capital loss on government debt. Capital loss on debt can occur due to either unexpected inflation or default. Expectations of capital loss raise the interest rate, and when the capital loss does not occur, debt accumulates in response to the higher interest rate. Equilibrium capital loss is never one hundred percent of debt, consistent with empirical evidence and in contrast to the traditional sovereign default model.

Optimization by the representative agent, together with the assumption that governments do not allow their debt to become negative in the limit,[13] implies a government intertemporal budget constraint given by[14]

$$\lim_{T\to\infty} E_t b_{t+T}\left(\frac{1}{1+r}\right)^T = (1+r)\,b_{t-1} - (\gamma_t - E_{t-1}\gamma_t) - E_t\sum_{k=0}^{\infty} s_{t+k}\left(\frac{1}{1+r}\right)^k = 0. \tag{4}$$

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[11] This ignores the effect of capital gains or losses on seigniorage revenue $\left(\frac{i_t}{1+i_t}\frac{\eta M_t}{P_t Y_t}\right)$ under the assumption that the fiscal authority can adjust the surplus to offset these.

[12] We assume growth is non-stochastic to simplify the analysis. We could analyze the implications of stochastic growth using a linearized model, but we reserve this for future work.

[13] Sims (1997), Woodford (1997) and Daniel (2001) argue that no country, acting to maximize utility of its own agents, would allow the present-value of its debt to become negative in the limit.

[14] Woodford (1994) derives the constraint as an equilibrium condition for a closed economy.

Note that unexpected capital loss on government debt is a source of government revenue.

2.4.2 Upper Bound

The **second key assumption** is that there is an upper bound on the present value of the primary surplus relative to output that a government can sustain. We motivate this with the realization that taxes are distortionary such that there is an upper bound on the fraction of income that a government can collect in taxes. Additionally, there is a limit below which government spending, necessary in the provision of public goods, cannot be reduced. Using the government's intertemporal budget constraint, equation (4), implies an upper bound on the current value of debt relative to output given by

$$b_t \leq \frac{\bar{\varphi}}{r}. \tag{5}$$

where $\bar{\varphi}$ is the value of the upper bound on the primary surplus relative to output. The upper bound rules out an equilibrium in which debt relative to output is explosive.[15] Since the model is specified in terms of the primary surplus and debt relative to output, we refer to these variables simply as the surplus and debt when there is no confusion.

2.4.3 Fiscal Policy Rule

We assume that the fiscal authority is able to commit to a policy rule for the surplus[16], in which the surplus responds to its own lag (s_{t-1}) and a linear combination of the target

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[15] This is more restrictive than an intertemporal budget constraint in the absence of an upper bound, in which debt can grow forever, as long as its growth rate is less than the interest rate, as in Bohn (1998, 2007). Additionally, Canzoneri, Cumby, and Diba (2001) base their empirical test determining whether monetary policy in the US is active or passive on the intertemporal budget constraint, as in early presentations of the FTPL. Sims (1997) argues that governments instead should be concerned with stabilizing debt relative to GDP, as in the current paper. Cochrane (1998) explains the difference in the two perspectives.

[16] The rule gives the government credibility, limiting the effect of negative fiscal shocks on the expected present value of future surpluses.

value of the long-run primary surplus (φ) and debt service (rb_{t-1}) at the growth-adjusted interest rate. The surplus rule for a particular country is given by

$$\begin{aligned} s_t &= (1-\alpha)\, s_{t-1} + \alpha \left[(1-\lambda)\, \varphi + \lambda r b_{t-1} \right] + \nu_t, & (6) \\ \frac{r}{1+r} &< \alpha < 1, \qquad 0 \le \lambda, \qquad 0 < \varphi \le \bar{\varphi}, \end{aligned}$$

where $(1-a)$ measures persistence in the surplus and λ represents the responsiveness of the surplus to the value for debt service relative to its long-run target value. The lagged value of the surplus reflects the desire to smooth the effect of shocks over time and is consistent with empirical evidence showing persistence in the primary surplus. The parameters α and λ are viewed as policy choices, and in the simulations we use the estimated values from Daniel and Shiamptanis (2010).[17] The **third key assumption** is that fiscal policy is subject to bounded, zero-mean stochastic shocks, ν_t. Stochastic shocks represent both truly unanticipated fiscal shocks, including war, natural disaster, or the recent financial crisis, as well as fiscal policy responses to the state of the economy.

Equations (3) and (6) imply dynamic equations for the surplus and debt

$$s_t = (1-\alpha)\, s_{t-1} + \alpha\, (1-\lambda)\, \varphi + \alpha \lambda r b_{t-1} + \nu_t \tag{7}$$

$$b_t = (1 + r - \alpha \lambda r)\, b_{t-1} - (1-\alpha)\, s_{t-1} - \alpha\, (1-\lambda)\, \varphi - \nu_t - \gamma_t + E_{t-1}\gamma_t \tag{8}$$

Together, the upper bound and stochastic shocks imply fiscal solvency risk to government debt, reflecting the reality that a government's commitment to raise taxes cannot be totally unconditional. Governments understand this risk, and the parameters they choose (α, λ) reflect their risk tolerance, determined, in part, by the cost of a crisis. Empirically,

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[17] The restrictions are imposed to yield a stationary long-run equilibrium in which debt and the surplus are not changing, relative to output.

countries do choose surplus rules with risk, and the Maastricht limits on debt and deficits reflect policy-maker concerns that at least some EMU countries might choose relatively risky rules.[18]

The **fourth key assumption** is that a solvent government honors its debt. Specifically, when the path of debt and the surplus, implied by its chosen fiscal policy and given in equations (7) and (8), keep debt from exceeding its upper bound, the sovereign honors its debt.

2.5 Stability and Dynamics

Stability properties are determined by the eigenvalues of the dynamic equations (7) and (8). Letting θ represent eigenvalues, which are assumed to be real and distinct, the characteristic equation for each country is given by

$$(1-\alpha)(1+r)-\theta\left[2+r(1-\alpha\lambda)-\alpha\right]+\theta^2=0. \tag{9}$$

Letting θ_1 be the larger eigenvalue, a value for $\lambda > 0$ is sufficient to assure that $\theta_1 < 1+r$, such that $\lim_{T\to\infty} E_t b_{t+T}\left(\frac{1}{1+r}\right)^T = 0$ assuring fiscal sustainability in the sense of Bohn (2007).

However, when $0 < \lambda < 1$, θ_1 exceeds unity, implying that debt relative to output can grow forever, eventually exceeding any upper bound. When debt has an upper bound, explosive paths for debt are inconsistent with equilibrium. The upper bound prevents a government from raising present-value surpluses sufficiently to service explosive debt.

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[18] The upper bound on debt and the fiscal policy rule together imply a lower bound on the surplus. There are values for the surplus so low that all values for debt violate the upper bound in the approach to the long-run equilibrium. Given an initial surplus value substantially above the minimum, the system is very unlikely to approach the minimum. However, in the event that it does, the error process must be restricted to rule out such low values.

Therefore, the fiscal policy parameters must imply global stability to rule out explosive paths. Global stability requires that both eigenvalues be on or inside the unit circle, which implies restricting λ such that $\lambda \geq 1$.

Effectively, with an upper bound, a necessary condition for fiscal solvency is that the fiscal authority follow a rule with $\lambda \geq 1$. This is stronger than fiscal sustainability in Bohn (2007). However, upper bounds can yield solvency crises even under a surplus rule with $\lambda \geq 1$, implying that this condition is not sufficient for solvency. A solvency crisis can occur if the adjustment path toward long-run equilibrium requires a value for debt which exceeds the upper bound. We turn to this below.

It is useful to represent the dynamics of the debt-surplus system using country phase diagrams.[19] We construct the phase diagram for each country by subtracting lagged values of the surplus from equation (7) and lagged values of debt from equation (8) to yield:

$$\Delta s_t = s_t - s_{t-1} = -\alpha s_{t-1} + \alpha \left(1 - \lambda\right) \varphi + \alpha \lambda r b_{t-1} + \nu_t, \tag{10}$$

$$\Delta b_t = b_t - b_{t-1} = \left(1 - \alpha \lambda\right) r b_{t-1} - \left(1 - \alpha\right) s_{t-1} - \alpha \left(1 - \lambda\right) \varphi - \nu_t - \gamma_t + E_{t-1}\gamma_t. \tag{11}$$

The phase diagram with $\lambda > 1$ and with $\nu_t = \gamma_t - E_{t-1}\gamma_t = 0$ is given in Figure 1. Debt service (rb) is on the vertical axis and the surplus is on the horizontal axis. The $\Delta s = 0$ and $\Delta b = 0$ schedules intersect at point P with $s_t = \varphi = rb_t$, representing a long-run equilibrium. The system is globally stable around its long-run equilibrium target values. If initial debt and the surplus are at point A, then the system is expected to travel along AP, eventually reaching the long-run equilibrium point P. Equations (10) and (11) can be used to show that with $\gamma_t - E_{t-1}\gamma_t = 0$, the relationship between debt and the surplus

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[19] Solutions for equations (7) and (8) with $\lambda \geq 1$ are given in the appendix.

along any adjustment path is given by

$$\frac{r\left(E_{t-1}b_t - b_{t-1}\right)}{E_{t-1}s_t - s_{t-1}} = r\left[\frac{rb_{t-1} - s_{t-1}}{\alpha\left(\lambda r b_{t-1} - s_{t-1} + (1-\lambda)\,\varphi\right) + E_{t-1}\nu_t} - 1\right]. \tag{12}$$

Note that in the range for which this slope is positive, a positive expected future fiscal shock reduces the slope of the adjustment path, such that debt is expected to attain lower values in its approach to a long-run equilibrium.

Over time, fiscal shocks (ν_t) could move the system away from its initial adjustment path, labelled AP, possibly to an adjustment path like HP. This path violates the government's intertemporal budget constraint because it requires that debt be expected to pass through a point where it exceeds the maximum present value of future surpluses. Since the fiscal authority could never service or repay such a large debt, agents could not expect to receive the market rate of return on debt along the path HP, implying that HP cannot be an equilibrium path.

As a country nears a crisis, which could require $\gamma_t > 0$, agents begin to anticipate capital loss. The expectation affects the evolution of debt and surpluses. Once shocks, together with expectations, send the system onto a path like HP, agents refuse to lend. This sudden stop of capital flows due to insolvency requires a fiscal response since the government cannot continue its policy of smoothing fiscal shocks using government debt. The timing of the sudden stop itself and the actual dynamics depend on how the fiscal authority is expected to react to the crisis. In this paper, we focus on the policy response of debt devaluation through default.

3 Fiscal Solvency Crisis

We model the equilibrium dynamics leading to a fiscal solvency crisis when the government responds to the crisis with default. We assume that agents know the fiscal response to the crisis.[20] A fiscal solvency crisis is most likely to occur in the region in which the debt and surplus are rising. We restrict initial values to this region. Equilibrium is defined below.

Definition 1 *Given constant values for the world interest rate and world price level, a monetary price-level target, a surplus rule (equation 7), an upper bound on debt (equation 5), and a policy-response in the event of a fiscal crisis for each country, an* ***equilibrium*** *is a set of time series processes for each country's primary surplus, debt, and capital loss on debt,* $\{b_t, s_t, \gamma_t\}_{t=0}^{\infty}$*, such that each government's flow and intertemporal budget constraints (equations 8 and 4) hold, expectations are rational, the debt for each country does not exceed its upper bound, and world agents expect to receive the return on assets determined by interest rate parity (equation 1).*

3.1 Default

Consider the case in which the country responds to a sudden stop of capital by reducing the magnitude of debt through default. With this crisis response, the fiscal authority remains committed to the fiscal policy rule, given by equation (10). As agents anticipate default in country h, $E_t\delta_{ht+1} < 1$. The monetary authority upholds its price level target by keeping $i_{jt} = i$ for all $j \neq h$, allowing i_{ht} to rise to satisfy equation (1) for the crisis country.

Assume that, when faced with a crisis in which it cannot borrow the desired amount, the government reduces the magnitude of debt through a default to assure that debt is not expected to travel above $\hat{\varphi}/\mathrm{r} \leq \bar{\varphi}/\mathrm{r}$ and to assure that it is expected to reach a long-run equilibrium value of $\frac{\varphi}{r}$. Note that we are allowing

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[20] Davig, Leeper, and Walker (2010) show that uncertainty regarding how a shock will ultimately be financed can affect dynamic behavior. Cooper, Kempf, and Peled (2008) show how alternative policy responses can represent multiple equilibria based on agents' beliefs about the policy response.

the government to choose a default magnitude larger than necessary to restore solvency, but we are assuming that agents know this choice. This requires that the government reduce the magnitude of the current value of debt, given by equation (8), to the value of debt along the adjustment path that is expected to reach a desired maximum at $\frac{\hat{\varphi}}{r}$. Note, that in contrast to earlier models of sovereign default, the magnitude of default will never be one hundred percent of debt.

Definition 2 *A* ***boundary locus*** *for debt service* (rb) *is located on the adjustment path tangent to the desired maximum value for* rb*, given by* $\hat{\varphi}$*. The boundary locus is the portion of this path for which the surplus is rising.*

Figure 1 shows the boundary locus for debt as BLXN. Debt service reaches its desired maximum value at point L, and debt service equals the surplus at point X. Note that the boundary locus is defined with respect to the government's desired maximum value of debt, not by its upper bound.

Given the government's policy response of reducing the value of debt to the boundary locus, the expectation of one-period-ahead capital loss on government debt is determined by the distance between the value of debt along the boundary locus, given by $\hat{b}_t$, and the current value of debt, given by b_t. We approximate the value for $\hat{b}_t$, implied by equations (10), (11), and (12), by taking a piecewise linear approximation of this path about s_{t-1} and $\hat{b}_{t-1}$, to yield[21]

$$\hat{b}_t = \hat{b}_{t-1} + \left(\beta_{t-1} - 1\right)\left(s_t - s_{t-1}\right), \tag{13}$$

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[21] The path for $\hat{b}_t$ has a maximum at $\frac{\hat{\varphi}}{r}$ and has $\gamma_t = E_{t-1}\gamma_t = 0$. We need its value for any given value for s_t. Therefore, we approximate its value at time t using its $t-1$ value $\left(\hat{b}_{t-1}\right)$ together with its $t-1$ slope and the change in the actual surplus $(s_t - s_{t-1})$.

where $\left(\beta_{t-1}-1\right)$ is the slope of the boundary locus BLXN,

$$\beta_{t-1}=\frac{r\hat{b}_{t-1}-s_{t-1}}{\alpha\left(\lambda r\hat{b}_{t-1}-s_{t-1}+(1-\lambda)\varphi\right)}, \tag{14}$$

and s_t-s_{t-1} is given by equation (10). The denominator in (14) represents the change in the surplus along the boundary locus and is always positive. Note that $\beta_{t-1}>1$ when debt is rising along BL, $\beta_{t-1}=1$ once debt reaches its maximum level at point L, and $\beta_{t-1}<0$ once the surplus exceeds debt service along XN, such that debt is falling. We show below that a positive crisis probability requires values of the surplus such that $\beta_{t-1}>0$.

We can compare the distance between $\hat{b}_t$ and b_t by subtracting equation (8) from equation (13) to yield

$$\Omega_t=\hat{b}_t-b_t=\mu_{t-1}\Omega_{t-1}+\beta_{t-1}\nu_t+\gamma_t-E_{t-1}\gamma_t, \tag{15}$$

where

$$\mu_{t-1}=1+\frac{\alpha r(1-\lambda)\left(\varphi-s_{t-1}\right)}{\alpha\left(\lambda r\hat{b}_{t-1}-s_{t-1}+(1-\lambda)\varphi\right)}>0.$$

The sign reflects the fact that adjustment paths, conditional on different initial values, do not cross.[22]

We define a shadow value of default, analogous to the shadow value of the exchange rate in generation one currency crisis models (Flood and Garber 1984). Conditional on a crisis in which agents refuse to lend, the shadow value of default represents the reduction in the value of debt needed for the economy to reach the boundary locus. The shadow value can be positive or negative.

Definition 3 *The* ***shadow value*** *of capital loss on debt due to default at time* t, $\tilde{\gamma}_t$, *is defined as the value of* γ_t *for which* $\Omega_t=0$.

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[22] It also reveals that the measure of the distance is a good approximation of the actual distance only in regions for which slope of the boundary locus BLXN is not changing too rapidly or for which the slopes of the boundary locus BLXN and the actual adjustment path are both positive.

Setting Ω_t to zero in equation (15) implies

$$\tilde{\gamma}_t = E_{t-1}\gamma_t - \mu_{t-1}\Omega_{t-1} - \beta_{t-1}\nu_t. \tag{16}$$

Assume that agents believe that the fiscal borrowing constraint will bind, creating default, if $\beta_{t-1} > 0$ and $\tilde{\gamma}_t > 0$**.** We prove that this assumption is consistent with a rational expectations equilibrium below in Proposition 2. Under this assumption, the actual value of capital loss due to default is given by

$$\gamma_t = \max\{\tilde{\gamma}_t, 0\} = \max\left\{E_{t-1}\gamma_t - \mu_{t-1}\Omega_{t-1} - \beta_{t-1}\nu_t, 0\right\}, \tag{17}$$

where we have used equation (16) to substitute for $\tilde{\gamma}_t$.

In order to solve for the magnitude of default, γ_t, we must first solve for the expectations of default, $E_{t-1}\gamma_t$.

Proposition 1 *Given a value of the surplus for which* $\beta_{t-1} > 0$ *and* $\mu_{t-1} > 0$*, together with the fiscal policy rule, with plans for default when the government cannot borrow, an equilibrium solution for the magnitude of expected default* $(E_{t-1}\gamma_t)$ *exists if and only if* $\Omega_{t-1} \geq 0$.

All proofs are contained in the appendix. Intuitively, the proposition implies that if debt is below its boundary locus at time $t-1$, creditors can be compensated for expectations of default. If debt is above its boundary locus at time $t-1$, there are no values for actual and rationally-expected period-t default, which both restore fiscal solvency and provide the market rate of return to international creditors.

Corollary 1 *For* $\beta_{t-1} > 0$*, when* $\Omega_{t-1} > 0$*, the probability of a crisis in period* t *is less than one, and when* $\Omega_{t-1} = 0$*, the probability of a crisis in period* t *is one.*

We can use the phase diagram in Figure 1 to understand expectations of default and the probability of a crisis. When the system is far from its boundary locus BLXN, such that no fiscal shock could send it over, the probability of a crisis next period is zero, implying that expectations of default are zero. The system is governed by the arrows of motion toward long-run equilibrium target values. Once the system reaches the neighborhood of the boundary locus, the probability of a crisis becomes positive and agents begin to expect default. The associated default-risk premium on debt increases the interest rate, causing debt to increase more quickly than shown along illustrated adjustment paths.

Once debt has risen so much that it lies on the boundary locus $(\Omega_{t-1} = 0)$, the probability of a crisis next period is unity. Therefore, expectations of default are so high that default is avoided only for the most favorable fiscal shock.[23] Using equation (17) to solve for the magnitude of default once $\Omega_{t-1} = 0$ yields

$$\gamma_t = E_{t-1}\gamma_t - \beta_{t-1}\nu_t \geq 0.$$

The sign restriction is necessary since default must be greater than or equal to zero for any realization of ν_t, including its upper bound value of $\bar{\nu}$. This yields

$$E_{t-1}\gamma_t \geq \beta_{t-1}\nu_t. \tag{18}$$

Therefore, when debt is on the boundary locus, there are multiple equilibria, in which actual and expected default must be positive and can be arbitrarily large. To verify, take the expectation of equation (17), when the probability of default is unity, to yield an identity in the expectation.

A value of $\Omega_{t-1} < 0$ would imply that debt is above the boundary locus, even with the most favorable fiscal shock, such that the probability of default is unity. Taking the

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[23] The probability of the most favorable shock in a continuous distribution is zero.

expectation of equation (17), when the probability of default is unity, yields

$$E_{t-1}\gamma_t = E_{t-1}\gamma_t - \mu_{t-1}\Omega_{t-1},$$

an impossibility. Rationally anticipated default cannot restore fiscal solvency because actual default cannot equal itself plus a gap. Therefore, in equilibrium, the dynamics must bound the system away from positions for which $\Omega_{t-1} < 0$. This criterion determines crisis timing.

Proposition 2 *There is no equilibrium without default in period t if* $\tilde{\gamma}_t > 0$*. Default, given by* $\gamma_t = \tilde{\gamma}_t$*, restores equilibrium.*

The proof in the appendix shows that if $\tilde{\gamma}_t > 0$ and there is no default, then $\Omega_t < 0$, violating conditions for an equilibrium. Therefore, default must occur whenever $\tilde{\gamma}_t > 0$ to assure equilibrium. Intuitively, in the event of a sudden stop, the country promises default in magnitude sufficient to place the system on the adjustment path toward the desired upper bound, thereby restoring fiscal solvency. The sudden stop occurs when $\tilde{\gamma}_t > 0$, and the government responds as promised. Therefore, Proposition 2 validates agents' assumption that the government will default whenever $\tilde{\gamma}_t > 0$.

Corollary 2 *A government which wants to sustain current fiscal policy as long as possible chooses* $\hat{\varphi} = \bar{\varphi}$*.*

A larger value for $\hat{\varphi}$ implies a higher boundary locus, implying that the distance between debt along the boundary locus and any initial value is greater. The greater is this distance, the lower is the probability of a fiscal solvency crisis.

Proposition 3 *In the absence of fiscal reform, equilibrium after default requires additional default each period until surplus rises above debt service.*

Initial default places the system on the boundary locus ($\Omega_{t-1} = 0$) implying that the expectations of default are large enough that default occurs next period for any fiscal shock. Post-crisis equilibrium is characterized by repeated default which can be arbitrarily large in magnitude. This is because of the one-sided nature of default, whereby default always reduces the value of debt. Expectations of default must be correct on average, implying that expectations of default must be the average value of default. Therefore, following the crisis, markets remain turbulent for some time. Agents expect additional default, interest rates are high, and additional default is necessary. This pattern does eventually end once the dynamics pass point X and move the economy toward its long-run equilibrium along the adjustment path.

Proposition 4 *Once the surplus is high enough that* $\beta_{t-1} \leq 0$, *the one-period-ahead probability of a crisis is zero.*

This is the region XN in Figure 1. Once $\beta_{t-1} \leq 0$, negative shocks do not send the system above the boundary locus, and positive shocks move the system further below the upper bound. Rapidly falling debt implies that the system is safe in this region.

3.2 Summary: Characteristics of a Sovereign Default Crisis

It is useful to summarize the characteristics of a fiscal solvency crisis under the assumption that it will be resolved with default to reduce the value of debt to a level consistent with fiscal solvency. First, a crisis generally occurs when debt is below its upper bound. There are two reasons for this. One is the upward sloping boundary locus in the approach to maximum debt, which implies that the value of debt along the boundary locus is lower for values of the surplus below the long-run equilibrium value. Second, a government might not be willing to let debt travel as high as its absolute maximum, effectively lowering the boundary locus and the value of debt which elicits a crisis.

A crisis is defined by a sudden stop of capital flows in which international creditors refuse to lend to the government. A government cannot borrow again in the private market until it has responded to the crisis in a way to entice creditors back into the market. Therefore, the government cannot postpone its response and continue its current fiscal policy when that policy requires either new debt or rolling over existing debt. This implies that lending is not restored until after the crisis-resolving policy of default. In particular, debt postponement or additional lending by an official agency like the IMF or the EU, without a reduction in the present value of repayments, cannot restore solvency and the ability to borrow on the private market.

Crises are imperfectly predictable. Once a crisis becomes possible, the interest rate rises, reflecting the expected capital loss on debt. The increase in the interest rate causes debt to accumulate more quickly, increasing the probability of a crisis. The more rapid growth in debt, due to the higher interest rate, implies that a crisis can occur even when the economy receives a favorable shock. This is possible when the favorable shock is small relative to the expected capital loss. However, if a country receives large enough favorable shocks, then it can escape the crisis.

Crises develop suddenly. For a country whose debt is substantially below the boundary locus, the probability of a crisis over a finite horizon is approximately zero. However, once debt is close enough to the boundary locus to elicit expectations of one-period-ahead capital loss, then rising interest rates increase the rate of growth of debt. This implies that to avoid a crisis, the country must on average receive favorable fiscal shocks.

Debt reduction, due to default, is never one hundred percent. The value for γ_t is determined in equilibrium to assure that debt reaches the boundary locus leading to a long-run equilibrium. Partial debt reduction in a crisis is consistent with empirical evidence, but it contrasts with the traditional sovereign default model in which debt reduction is always one hundred percent.

3.3 Other Possible Policy Responses

Reduction of the magnitude of debt through default is not the only possible policy response to a fiscal solvency crisis. Other possible responses are briefly considered here, but full analysis of them is in other papers (Daniel 2010, Daniel and Shiamptanis 2010) or is left to future research.

Once a crisis becomes anticipated with positive probability, the government could implement fiscal reform, designed to raise the expected present value of future surpluses. The reform could take the form of changes in the policy parameters of the surplus rule (α, λ) or an increase in the surplus for a specific period of time, as with a positive mean to the fiscal shock, ν_t. If this policy response raises the expected present value of future surpluses enough to eliminate near-term risk, and agents know that fiscal reform will be the policy response to a sudden stop in lending, then there should be no increase in a country's interest rate as it nears a crisis. However, given that the probability of a fiscal solvency crisis becomes positive following negative fiscal shocks, most likely accompanied by recession, the promise of larger near-term surpluses is unlikely to satisfy a cost-benefit analysis. The sovereign must weigh the costs and benefits of fiscal austerity in recession, when it had not chosen austerity in normal times. A decision not to undertake fiscal reform has features similar to the original sovereign default models. It is quite likely that a sovereign, facing a fiscal solvency crisis and a recession, would decide against additional austerity, and that is implicitly the assumption in our model with default as the policy response to a crisis. Additionally, if the sovereign does choose fiscal reform, its effectiveness in keeping interest rates from rising in the run-up to the crisis and restoring lending after the crisis, depends on its credibility. Austerity in bad times could face credibility issues, although IMF or European Union oversight might enhance credibility. However, even after fiscal reform, fiscal policy still has risk, and future shocks could once again raise the probability of a crisis.

Another possible response would be the promise of fiscal transfers from member countries to the crisis country. Such a response was negotiated between Greece and the EMU in April 2010 in the form of a loan with below-market interest rates. If the market knew that the EMU would provide fiscal transfers sufficient to restore solvency without default, then interest rates should not rise as debt nears the boundary locus. However, such transfers explicitly violate the EMU agreement whereby countries are not liable for debts of member countries. And accepting liability in the event of a crisis has an obvious moral hazard problem. The fact that interest rates have risen inspite of the commitment for the loan, implies that the subsidy is not large enough to move Greece far enough below the boundary locus to eliminate expectations of default.

Alternatively, the union's monetary authority could use monetary policy instruments to restore fiscal solvency to the insolvent government. It could resort to an increase in traditional seigniorage to provide additional revenue to the crisis country, effectively increasing feasible surpluses and the upper bound on debt. Or it could switch to a passive monetary policy, allowing the crisis country to switch to an active fiscal policy, as in the Fiscal Theory of the Price Level, whereby stochastic and symmetric jumps in prices maintain fiscal solvency. Such a policy switch is usually accompanied by a jump in inflation on the crisis date, reducing the real value of debt through surprise inflation. Daniel (2010), Daniel and Shiamptanis (2010) provide an analysis of this policy response.

An extreme response would be withdrawal by the crisis country from the monetary union and re-issuance of its own currency, as suggested by Sims (1999), thereby restoring its on monetary policy instruments to assure solvency. This policy must be accompanied by either an increase in traditional seigniorage or by policy switching.

Finally, it is useful to consider how uncertainty about the nature of the policy response would affect crisis dynamics, which is determined largely by the interest response to the expectations of a crisis. Consider the case of Greece and other highly indebted EMU countries, and assume that the market places a positive probability on both default and on fiscal transfers from the EMU. The positive probability on fiscal transfers should restrain the rise in interest rates as debt approaches the boundary locus. Now, assume that the EMU announces, counterfactually, that it will provide no fiscal transfers to Greece. Greek interest rates would rise, to reflect the fact that all the probability mass has moved to default. Additionally, interest rates in other highly-indebted EMU countries would rise to reflect the same change in expectations, increasing the probability of additional crises. Hence, the EMU justifies the fiscal transfers to Greece is as a policy to prevent contagion and additional crises.

4 Simulations of Crisis Risk

In this section, we use simulations to quantify the fiscal solvency risk faced by different countries in a monetary union when the crisis country responds with default. Given parameter values for the N surplus rules, the distribution of ν_t, and the method of crisis resolution, the system can be solved numerically and simulated to quantify the risk of one country in the N-country monetary union encountering a crisis over a given period of time.

For the simulations, we use estimates for the parameters of the surplus rule from Daniel and Shiamptanis (2009). They provide group mean estimates of parameters for the surplus rule in real terms using cointegration and error-correction models for a panel of ten EMU countries with annual data over the 1970-2006 period. The baseline parameters we use for the simulations are reported in Table 1.

Table 1: Baseline Parameters

	i	α	λ	g
parameters	0.0422	0.4987	1.3003	0.0262
standard errors	0.0061	0.0717	0.0901	0.0027

We adjust these estimates by the group mean panel estimate of the long-run value of real output growth g to provide estimates for the parameters expressed as a fraction of output.[24] These parameters imply a growth-adjusted interest rate given by $r = \frac{i-g}{1+g} =$ 0.0156. For the target value of the long-run primary surplus, φ, we use the value of

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[24] The variables in the paper of Daniel and Shiamptanis (2009) are in real terms, whereas the variables in this paper are expressed as percentages of output. This implies that the α in this paper is $\alpha = \frac{0.5118}{1+g} =$ 0.4987.

0.93% of GDP which implies a target value for the long-run debt/GDP of 60%. Under the assumption that fiscal shocks have a normal distribution with mean zero, the panel estimate of their standard error is 1.42% of GDP. We let the upper bound on the fiscal shocks, $\bar{\nu}$, be 4.26% of GDP, which corresponds to three standard deviations. We set the desired maximum value of debt, $\frac{\hat{\varphi}}{r}$, which is the effective upper bound, at 141%, larger than any of these countries has experienced the last thirty-six years.

We use 1,000 replications of a ten-year simulation to estimate the probability of a fiscal crisis. In each simulation, initial values of debt/GDP (b_{t-1}) and the primary surplus/GDP (s_{t-1}) are used to set the initial value for the distance, Ω_{t-1}, and for the slope of the boundary locus, $\beta_{t-1} - 1$. If $\beta_{t-1} > 0$, then based on Ω_{t-1}, the critical value for the shock that would elicit default (ν_t^*) and the expectation for capital loss ($E_{t-1}\gamma_t$) are calculated. The dynamic system then receives a fiscal shock (ν_t) from the truncated normal distribution and the value for capital loss (γ_t) is computed. If $\beta_{t-1} > 0$ and $\gamma_t = 0$, or if $\beta_{t-1} < 0$, then next period's surplus and debt are updated using equations (7) and (8), which are then used to update Ω_t and β_t.[25] The process is repeated for ten years. If during the ten-year simulation we have a value of $\gamma_t > 0$, then there is a crisis and the simulation ends. We repeat the ten-year simulation 1000 times. The probability of a crisis over ten-years is the number of crises divided by 1000, the number of replications.

To determine the safety of a country which adheres to the Maastricht rules, we simulated the model with values for initial debt and the primary surplus equal to the upper bound of the Maastricht limits. We set debt at 60% of GDP and the primary surplus at -2.06% which implies an actual surplus of -3% of GDP. Under the baseline parameter

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[25] We also check that $\mu_{t-1} > 0$. In our simulations, we never obtain a negative value.

values, fiscal policy is completely safe with no crises over ten years in the 1,000 replications. We considered several sensitivity analysis scenarios to raise the risk. These include changing parameter values one at a time by one standard deviations in the risky direction.[26] The probability of a fiscal crisis for a country at the Maastricht limits is zero under all sensitivity analyses designed to increase risk. Therefore, these results suggests that countries with debt and primary surplus at the Maastricht limits are perfectly safe over the ten year horizon.

Next we consider whether countries like Belgium, France and Germany, with moderate deviations from the Maastricht limits, are moving into a risky region. We repeated the simulations for these countries, using their 2009 values of debt/GDP and primary surplus/GDP.[27] Under the baseline parameters Belgium, France and Germany are perfectly safe over the ten-year horizon. Additionally, these countries are perfectly safe under all one-standard-deviation parameter changes except for the changes which increase the real interest rate (i) by 61 basis points or a reduce real output growth rate (g) by 27 basis points. The increase in i raises the probability of a crisis for Belgium to 100%, for France to 8.1%, and for Germany to 0.1%. A reduction in g raises the probability of a crisis only for Belgium to 2.3%. Since the probability of a crisis rises so dramatically, especially for Belgium, we calculated how large r would have to be for Belgium, France and Germany to begin experiencing risk. In Figure 2, we show that Belgium experiences positive risk with a 17 basis points increase in r. France and Germany require larger increases of 37 and 59 basis points, respectively. Therefore, deviations from the Maastricht limits could

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[26] Experiments included raising i to 0.0483, reducing λ to 1.2102, reducing α to 0.427, and reducing g to 0.0235

[27] The 2009 values for debt/GDP and primary surplus/GDP for Belgium were 101.18% and -2.09%, for France were 84.53% and -5.28%, and for Germany were 77.36% and -0.94%. Source: OECD March 2010.

imply risk for parameter values of variables, determining the real interest rate, within one standard deviation of their estimated values.

Next we consider whether high-debt countries like Italy and Greece, which have violated the Maastricht rules, face risk over the next ten years. For Italy, the 2009 value of debt/GDP was 123.57%, and the primary surplus/GDP was -0.60%. For Greece, the 2009 value of debt/GDP was 114.88% and the primary surplus/GDP was -8.17%.[28] Under the baseline parameter values, the probability of crisis for Italy is 2.3% and for Greece is 1.4%. Of course, small changes in the parameters in the risky direction increase risk, with the risk increasing at an increasing rate due to the effect of expectations. How would less risky values of parameters affect risk? We can think of these changes as fiscal reform which could be taken to reduce the probability of a crisis as it approaches.

To address this question, we consider how the crisis probability changes when the policy parameters α and λ change by one standard deviations in the less-risky direction. When λ increases to 1.3904, which implies that primary surplus responds more strongly to debt, the probability of a fiscal crisis for Italy falls to 1.4% and for Greece falls to 0.9%. When α increases to 0.5704, which implies less persistence in the primary surplus, the probability of a fiscal crisis for Italy falls to 0.8% and for Greece falls to 0.1%. The results in Figure 3 show that the probability of fiscal crisis is sensitive to changes in α and λ, but even if countries choose less-risky policy parameters by a full standard deviation, they cannot completely eliminate risk. The risk is due to the upper bound and the stochastic shocks to the surplus.

These estimates of risk are based on a maximum allowable value of debt $\left(\frac{\hat{\varphi}}{r}\right)$ of 141%

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[28] Source: OECD Economic Outlook March 2010.

of GDP. Perhaps risk is lower because Greece and Italy would actually be willing and able to tolerate a higher value for debt. Therefore, we consider how high the effective upper bound $\left(\frac{\hat{\varphi}}{r}\right)$ would have to be for Italy and Greece to be safe. Under baseline parameters, Figure 4 shows that crisis probability becomes zero for both countries once $\frac{\hat{\varphi}}{r}$ increases to 148%. Numerical computation shows that a fiscal policy with debt reaching 148% must allow primary surpluses to reach values of 2.67%. In the last forty years, primary surplus for Italy was higher than 2.67% 8 times, and for Greece was higher than 2.67% 5 times.[29] In contrast, Figure 4 shows that slightly lower values for maximum debt $\left(\frac{\hat{\varphi}}{r}\right)$ increase crisis probability dramatically.

OECD projects debt to rise in 2011 for many European countries. Therefore, we considered how high debt would have to be for Belgium, France and Germany to begin experiencing risk under baseline parameter values, and how crisis probability changes for Italy and Greece as debt increases from its 2009 value. Using baseline parameter values with the primary surplus at its 2009 value, Figure 5 shows that crisis probability becomes positive for Belgium, France and Germany once debt exceeds 114%, 111% and 115% of GDP, respectively. All of these exceed OECD forecasts for 2011[30], implying no risk of a crisis at baseline parameter values. The OECD forecasts 2011 debt for Italy at 129.7% and for Greece at 130.2%. If debt does reach these levels with surpluses at their 2009 values, then the ten-year risk of a fiscal crisis for Italy rises to 10.4% and for Greece to 69.1%. The large increase in the probabilities reflects the result that the probability of a crisis is increasing in debt at an increasing rate.

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[29] In the last forty years, the largest primary surplus for Italy is 6.09% (in 1997) and for Greece is 3.65% (in 1999).

[30] OECD forecasts for debt for Belgium, France and Germany are 108.45%, 99.15% and 85.46% of GDP, respectively. (OECD Economic Outlook March 2010)

In addition, Figure 5 shows that with a maximum allowable value of debt $\left(\frac{\hat{\varphi}}{r}\right)$ of 141%, the crisis probability becomes positive for Greece when debt exceeds 109% of GDP. This shows that a fiscal solvency crisis can occur when debt is 32 percentage points below its effective upper bound. This is because of the upward sloping boundary locus towards the maximum debt which implies that the upper bound is lower for small values of primary surplus.

Crisis probability rises at an increasing rate, either as parameter values change in the risky direction or as debt increases, because expectations of debt devaluation rise as the economy approaches the boundary locus. Higher expected debt devaluation increases the interest rate and increases the rate at which debt accumulates. As a country approaches the boundary locus, a slight change in parameters or in debt can create a dramatic change in crisis probability. This illustrates forcefully that a country receiving favorable shocks can substantially reduce and/or eliminate the probability of crisis without fiscal reform. It also illustrates the reverse. A country can substantially increase its crisis probability with small changes in debt which push it critically toward the boundary locus.

5 Conclusions

What does economic theory have to say about recent interest rate spikes in Greece, and the Greek government and the EMU focus on fiscal measures to resolve the problem? Existing sovereign default models, either those in which the sovereign weighs the costs and benefits of complete default or those in which default is the bad outcome in a model with multiple equilibria, are not designed to address the fiscal solvency issues that appear to be at the heart of this potential crisis.

The first contribution of this paper is the development of a new model of sovereign default. Monetary union eliminates individual-country monetary policies, which could be used to provide seigniorage and debt devaluation through inflation, leaving a country more vulnerable to fiscal insolvency. We assume that institutions are strong enough that a sovereign usually repays, but no sovereign can make an unconditional commitment to repay. We model this by assuming that the fiscal authority commits to a surplus rule with parameter values chosen to reflect the optimal choice of risk. The surplus rule is subject to stochastic shocks, as with the recent financial crisis. Together the stochastic shocks and the upper bound on debt imply that the government could find itself in a position for which the expected present value of future surpluses is less than the value of debt necessary to continue the fiscal policy rule. Agents refuse to lend into this position of insolvency, and a sudden stop of capital flows defines the fiscal solvency crisis. Once crisis probability becomes positive, interest rates increase, reflecting expectations of capital loss on debt. Rising interest rates accelerate the rate of increase of debt, increasing the probability of a crisis at an increasing rate.

The second contribution of this paper is that we simulate the model to provide estimates of the probability of a fiscal solvency crisis for EMU countries, under the assumption that a crisis will be resolved with default. We find that for countries with debt and the primary surplus bound by the Maastricht Treaty limits, there is no probability that stochastic fiscal shocks could send debt onto a path along which it would be expected to violate the upper bound. Therefore, countries adhering to the Maastricht limits have no probability of fiscal solvency crisis over a ten year horizon. We also simulate the model using 2009 values of debt and the primary surplus for several EMU countries. We find that countries with moderate violations of the Maastricht rules are safe, while countries with large violations, like Greece and Italy, are not.

We can use these results and the implications of the model to understand the early 2010 interest rate spikes in Greece as reflecting expectations of a fiscal solvency crisis. The model implies that the high government debt and the low primary surplus, together with the possibility that a fiscal solvency crisis could be resolved with default, were the cause of the interest rate premium. News that the Greek government misrepresented the value of its surpluses reduced the expected present value of future surpluses and increased this interest rate premium. Non-concessionary loans from the EMU can reduce imminent borrowing needs from the private sector, but since they do not either reduce the value of debt or raise the expected present value of future surpluses, they have no impact on the probability of a future solvency crisis. Concessionary loans or outright transfers from the EU could reduce the probability of a fiscal solvency crisis. Greek promises in March 2010 to raise current and near-term primary surpluses were successful in bringing down the interest premium and did allow the government to sell bonds on the private market. However, the interest rate premium remains positive as of this paper's writing, reflecting a continued positive risk of a fiscal solvency crisis. The model implies that unless transfers and/or present-value surplus adjustments are large enough to set the probability of crisis in the near-term to zero, Greece must experience fiscal shocks more favorable than average to avoid a fiscal solvency crisis.

6 Appendix: Default

6.1 Solutions

When fiscal policy is passive and monetary policy active, the time paths for each country's surplus and debt relative to output are

$$\begin{aligned} s_t &= \varphi + \frac{(\theta_2 - 1 + \alpha)\,\theta_1^t}{(1-\alpha)(\theta_1 - \theta_2)} \left\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\ &\qquad \left. + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \\ &\quad + \frac{(\theta_1 - 1 + \alpha)\,\theta_2^t}{(1-\alpha)(\theta_1 - \theta_2)} \left\{ (1 - \alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\ &\qquad \left. + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \end{aligned} \tag{19}$$

$$\begin{aligned} b_t &= \frac{\varphi}{r} + \frac{\theta_1^t}{\theta_1 - \theta_2} \left\{ (\alpha - 1)(s_0 - \varphi) + (\theta_1 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\ &\qquad \left. + \sum_{k=1}^{t} \theta_1^{-k} \left[-\theta_1 \nu_k - (\theta_1 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \\ &\quad + \frac{\theta_2^t}{\theta_1 - \theta_2} \left\{ (1 - \alpha)(s_0 - \varphi) - (\theta_2 - 1 + \alpha)\left(b_0 - \frac{\varphi}{r}\right) \right. \\ &\qquad \left. + \sum_{k=1}^{t} \theta_2^{-k} \left[\theta_2 \nu_k + (\theta_2 - 1 + \alpha)(\gamma_k - E_{k-1}\gamma_k) \right] \right\} \end{aligned} \tag{20}$$

where $\theta_1 \leq 1$ and $\theta_2 < 1$ are the eigenvalues of the characteristic equation (9). When the country is far from a crisis, $\gamma_t = E_{t-1}\gamma_t = 0$. The values for γ_t and its expectations in the neighborhood of a crisis are endogenized as part of the model's full solution.

6.2 Proofs

6.2.1 Proof of Proposition 1

Assume a value for s_{t-1} for which $\beta_{t-1} > 0$ and $\mu_{t-1} > 0$, and define $f(\nu_t)$ as a bounded, symmetric, mean-zero distribution for ν_t, with bounds $\pm\,\bar{\nu}$. Define ν_t^* as a critical value for ν_t such that

$$\begin{aligned}\gamma_t &> 0 \text{ for } \nu_t < \nu_t^*, \\ \gamma_t &= 0 \text{ for } \nu_t \geq \nu_t^*.\end{aligned}$$

When such a critical value exists, taking the expectation of equation (17) yields

$$E_{t-1}\gamma_t = \int_{-\bar{\nu}}^{\nu_t^*} \gamma_t f(\nu_t)\, d\nu_t = \int_{-\bar{\nu}}^{\nu_t^*} \left[E_{t-1}\gamma_t - \mu_{t-1}\Omega_{t-1} - \beta_{t-1}\nu_t\right] f(\nu_t)\, d\nu_t. \tag{21}$$

Defining $F(\nu_t^*)$ as the cumulative at ν_t^* and collecting terms on the expectation yields

$$\left[1 - F(\nu_t^*)\right] E_{t-1}\gamma_t = -\mu_{t-1}\Omega_{t-1} F(\nu_t^*) - \beta_{t-1} \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t. \tag{22}$$

Substituting into equation (17) yields an implicit expression for γ_t as

$$\left[1 - F(\nu_t^*)\right]\gamma_t = \max\left\{0, -\left[\mu_{t-1}\Omega_{t-1} + \beta_{t-1}\int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \beta_{t-1}\left[1 - F(\nu_t^*)\right]\nu_t\right]\right\}, \tag{23}$$

where $F(\nu_t^*)$ has the interpretation as the probability of fiscal solvency crisis.

To determine the probability of fiscal solvency crisis, $F(\nu_t^*)$, and the expectations of default, $E_{t-1}\gamma_t$, we first solve for ν_t^*. Define $\chi_t = \int_{-\bar{\nu}}^{\nu_t^*} \nu_t f(\nu_t)\, d\nu_t + \left[1 - F(\nu_t^*)\right]\nu_t^*$. A solution for ν_t^* exists iff there exists a value for ν_t^*, satisfying $-\bar{\nu} \leq \nu_t^* \leq \bar{\nu}$, which sets $\mu_{t-1}\Omega_{t-1} + \beta_{t-1}\chi_t = 0$ such that $\gamma_t = 0$ in equation (23).

Given that $\beta_{t-1} > 0$ and $\mu_{t-1} > 0$, the proof must show that $\chi_t \leq 0$ for all possible values for ν_t^*. Let ν_t^* take on its smallest possible value of $-\bar{\nu}$. Then $\chi_t = -\bar{\nu} < 0$. The derivative of χ_t with respect to ν_t^* is given by $1 - F(\nu_t^*)$. For $\nu_t^* < \bar{\nu}$, the derivative is positive. Therefore, as ν_t^* rises, χ_t rises monotonically. Once ν_t^* takes on its largest possible value, given by $\bar{\nu}$, $1 - F(\bar{\nu}) = 0$, and χ_t takes on its maximum value of zero.

Therefore, $\chi_t \leq 0$ for all feasible values of ν_t^*. Since $\chi_t \leq 0$, a necessary and sufficient condition for $\mu_{t-1}\Omega_{t-1} + \beta_{t-1}\chi_t = 0$ is $\Omega_{t-1} \geq 0$.

When $\Omega_{t-1} \geq 0$, a solution for ν_t^* exists, and the expectations of default are given by the solution of equation (22).

6.2.2 Proof of Corollary 1

When $\Omega_{t-1} > 0$, $\chi_t < 0$, requiring $\nu_t^* < \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\nu_t^*)$, is less than one. When $\Omega_{t-1} = 0$, ν_t^* must set $\chi_t = 0$, implying that $\nu_t^* = \bar{\nu}$. Therefore, the probability of a crisis, given by $F(\bar{\nu})$, is one.

6.2.3 Proof of Proposition 2

Equilibrium in period t requires $\Omega_t \geq 0$. This is because Proposition 1 shows that there can be no rational expectations value for $E_t\gamma_{t+1}$ when $\Omega_t < 0$ under the initial policy mix without default. Therefore, if $\Omega_t < 0$, there is no equilibrium unless the country defaults. Using equation (15) and (16), yields

$$\Omega_t = \mu_{t-1}\Omega_{t-1} + \beta_{t-1}\nu_t - E_{t-1}\gamma_t + \gamma_t = \gamma_t - \tilde{\gamma}_t.$$

Therefore, when $\Omega_t < 0$, $\tilde{\gamma}_t > 0$. A positive shadow rate triggers default. Default, with $\gamma_t = \tilde{\gamma}_t$, sets $\Omega_t = 0$, restoring equilibrium by Proposition 1.

6.2.4 Proof of Corollary 2

The position of the boundary locus is increasing in $\hat{\varphi}$.

6.2.5 Proof of Proposition 3

A default in period t, which brings the system to the boundary locus, implies that $\Omega_t = 0$. When $\Omega_t = 0$, the probability of a crisis in period $t+1$ is unity by Corollary 1, and Proposition 1 yields $E_t\gamma_{t+1} \geq \beta_t\bar{\nu}$. Given a realization for $\nu_{t+1,}$ default occurs in the magnitude to set $\Omega_{t+1} = 0$. The pattern persists until the dynamics imply that debt passes point X along the adjustment path BLP.

6.2.6 Proof of Proposition 4

The calculations in the proof of Proposition 1 can be used to demonstrate that the expected value of default is zero once $\beta_{t-1} = 0$, implying that the one-period-ahead probability of a crisis is zero. Once $\beta_{t-1} < 0$, equation (15) shows that negative shocks increase the distance such that crises do not occur for negative shocks. Positive shocks reduce the distance, but since they also reduce debt, they cannot send debt above $\frac{\hat{\varphi}}{r}$. Therefore, if the system has not traveled above the boundary locus, as necessary in equilibrium, and has reached a point such that $\beta_{t-1} < 0$, a shock cannot send debt above its desired maximum level from this position. Crisis probability is zero.

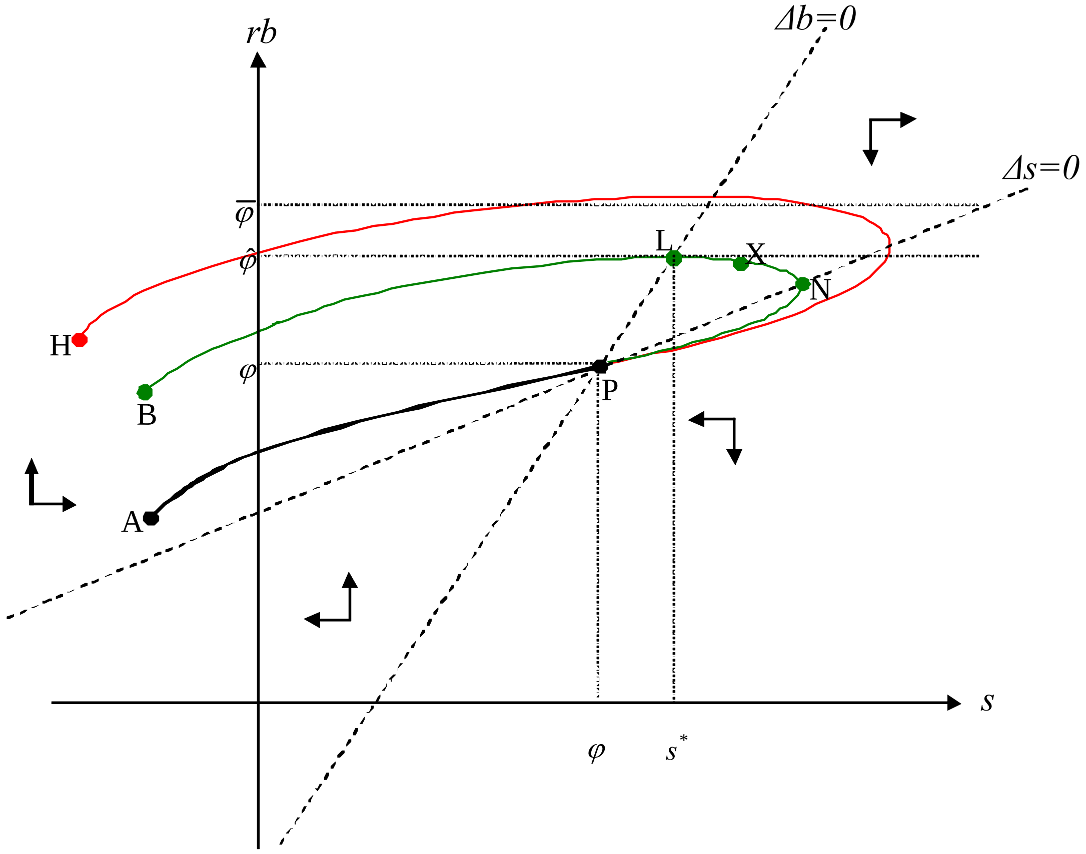

Figure 1: Passive Fiscal Policy

Note: $s^* = \frac{\hat{\varphi}(1-\alpha\lambda)-\alpha(1-\lambda)\varphi}{1-\alpha}$ is the value of s along the adjustment path BP at the point L with $rb = \hat{\varphi}$.

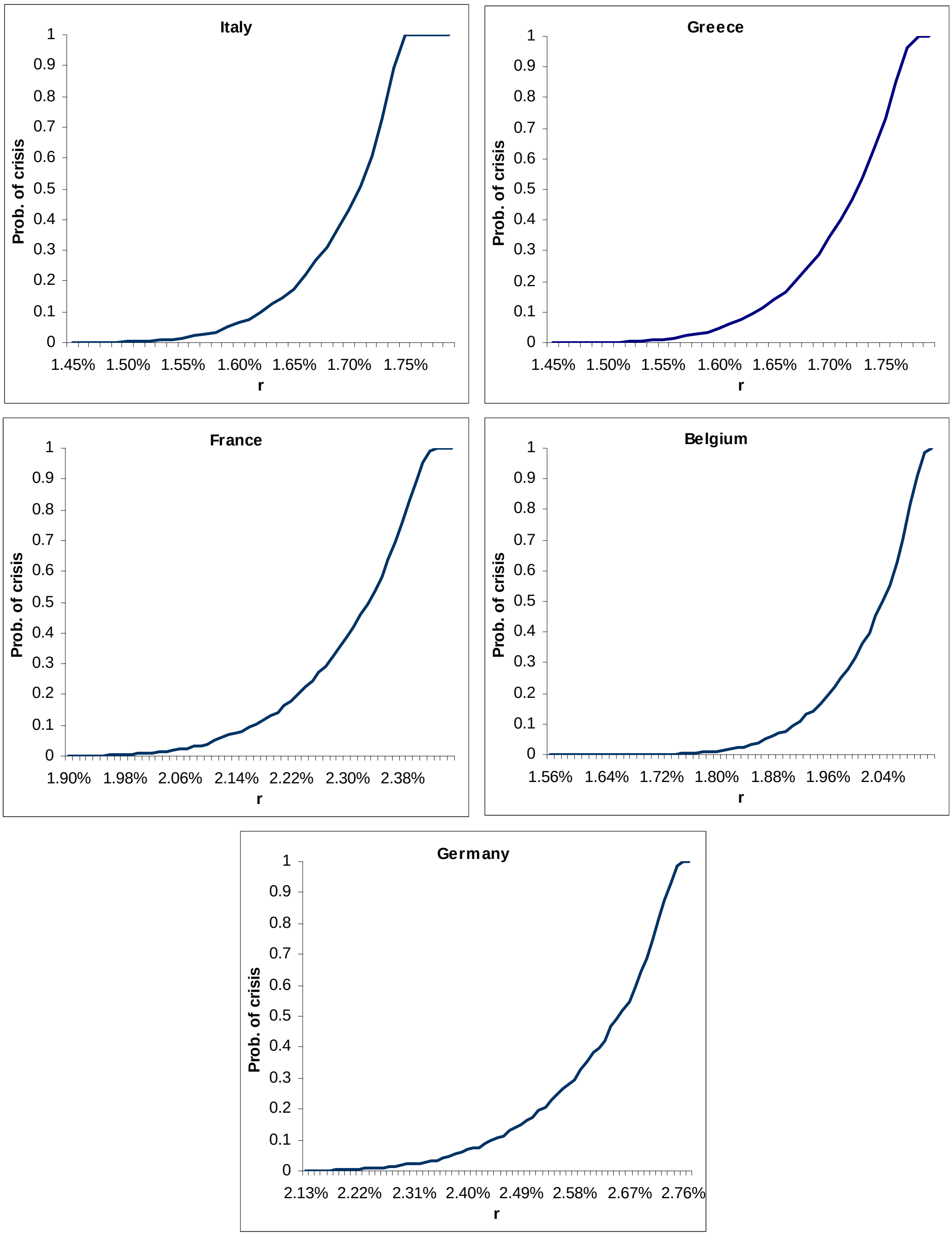

Figure 2: The probability of a crisis as a function of growth-adjusted real interest rate.

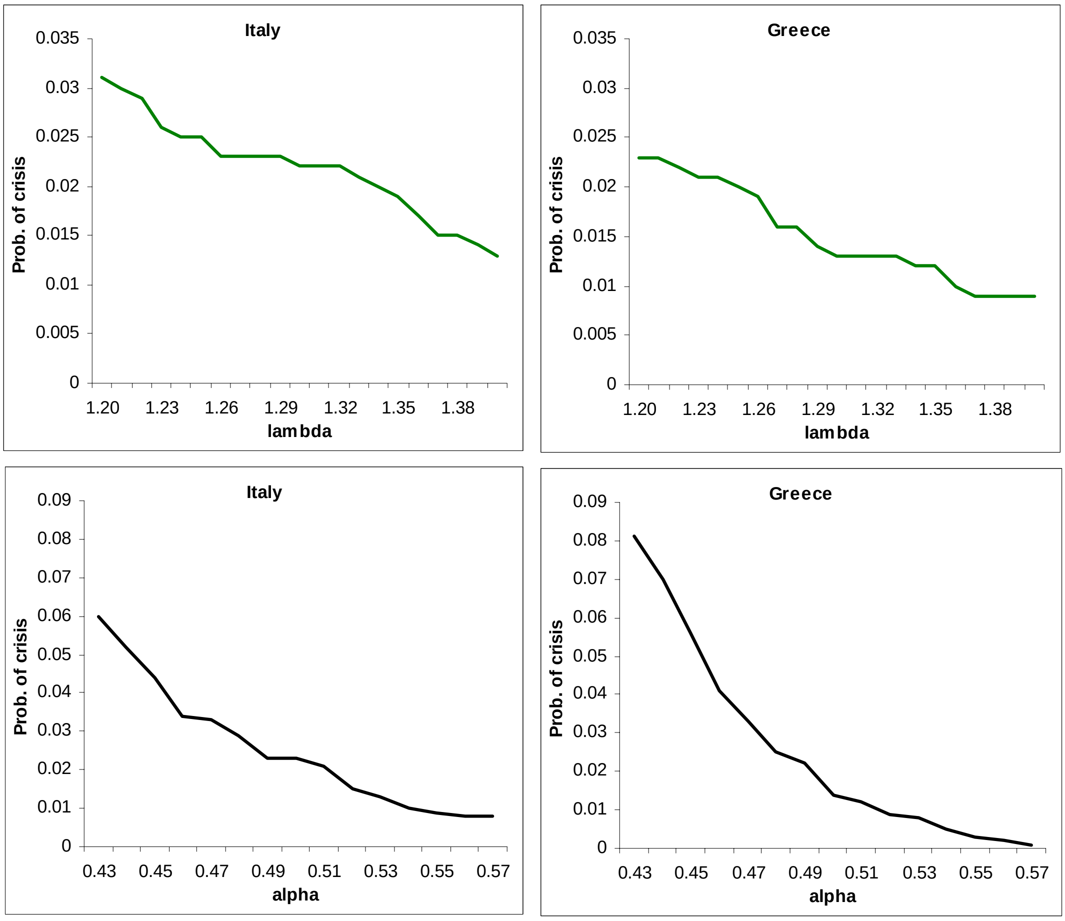

Figure 3: The probability of fiscal crisis as a function of lambda and alpha.

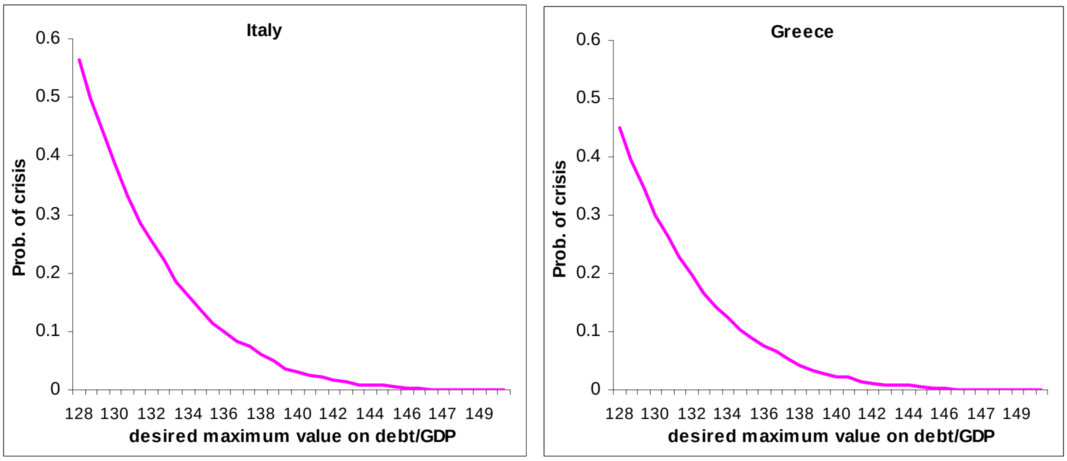

Figure 4: The probability of fiscal crisis as a function of the desired maximum value of the debt/GI

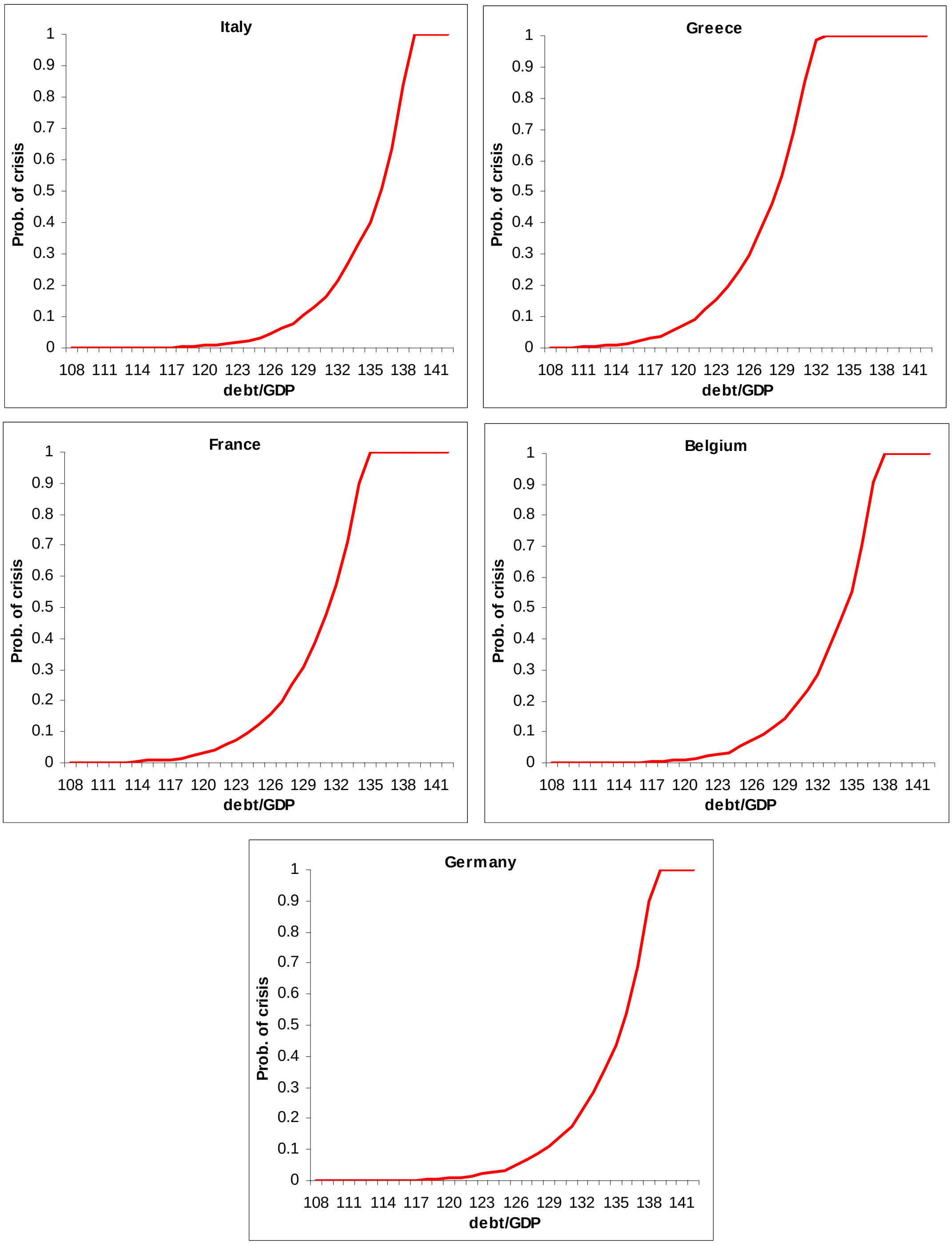

Figure 5: The probability of fiscal crisis as a function of debt/GDP

References

[1] Arellano, Cristina and Johathan Heathcote, “Dollarization and Financial Integration.” (2009) forthcoming *Journal of Economic Theory.*

[2] Bi, Huixin, "Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy. (2010), manuscript, University of Indiana.

[3] Bi, Huixin and Eric Leeper, "Sovereign Debt Risk Premia, Fiscal Limits, and Fiscal Policy." (2010), manuscript, University of Indiana.

[4] Bohn, H., "The Behavior of U.S. Public Debt and Deficits," *Quarterly Journal of Economics*, 113, (1998), 949-963.

[5] Bohn, H., "Are Stationarity and Cointegration Restrictions Really Necessary for Intertemoral Budget Constraints?", *Journal of Monetary Economics* 54, (2007), 1837-1847.

[6] Canzoneri, M., R. Cumby, and B. Diba, "Is the Price Level Determined by the Needs of Fiscal Solvency?" *American Economic Review*, 91(5), (2001), 1221-1238.

[7] Chari, V.V, Lawrence J. Christiano, and Patrick Kehoe, "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit, and Banking 23* (August 1991), 519-539.

[8] Cochrane, J., "A Frictionless View of U.S. Inflation," in Ben Bernanke and Julio Rotemberg, eds., *NBER Macroeconomics Annual* (Cambridge, MA: MIT Press 1998), 323-384.

[9] Cole, Harold and Timothy Kehoe, "A Self-fulfilling Model of Mexico’s Debt Crisis. "*Journal of International Economics* 41(3/4) (November 1996).

[10] Cole, Harold and Timothy Kehoe, " Self-fulfilling Debt Crises." *Review of Economic Studies* 67(1) #230, (2000) 91-116.

[11] Cooper, Russell, Hubert Kempf, and Dan Peled. "Regional Debt in Monetary Unions: Is it Inflationary?" CES Working Waper (December 2008).

[12] Daniel, Betty C., "Exchange Rate Crises and Fiscal Solvency," (2010) forthcoming *Journal of Money, Credit, and Banking.*

[13] Daniel, Betty C. “The Fiscal Theory of the Price Level in an Open Economy.” *Journal of Monetary Economics* 48 (October 2001a), 293-308.

[14] Daniel, Betty C. “A Fiscal Theory of Currency Crises.” *International Economic Review* 42 (November 2001b) 969-988.

[15] Daniel, Betty C. and Christos Shiamptanis, "Fiscal Policy in the European Monetary Union," working paper, University at Albany, (2009).

[16] Daniel, Betty C. and Christos Shiamptanis, "Fiscal Risk Assessment: Canada", working paper, University at Albany, (2010).

[17] Davig, Troy, Eric Leeper, and Todd B. Walker, "’Unfunded Liabilities’ and Uncertain Fiscal Financing." (2010) manuscript, University of Indiana.

[18] Eaton, Jonathan and Mark Gersovitz. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *The Review of Economic Studies* 48(2) (April 1981), 289-309.

[19] Eaton, Jonathan and Raquel Fernandez. "Sovereign Debt." In Gene M. Grossman and Kenneth Rogoff, eds., *Handbook of International Economics*, vol 3. Amsterdam: North Holland (1995).

[20] Flood, R. P. and P. M. Garber, "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics* 17 (August 1984), 1-13.

[21] Garcia, Marcio and Roberto Rigobon, "Risk Management Approach to Emerging Market's Sovereign Debt Sustainability with an Application to Brazilian Data." NBER Working paper 10336 (March 2004).

[22] Sargent, Thomas and Neil Wallace, "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Quarterly Review* (1981).

[23] Sims, Christopher A. "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy." *Economic Theory* 43(3) (1994), 381-399.

[24] Sims, Christopher A. "Fiscal Foundations of Price Stability in Open Economies." Working Paper, Yale University (1997).

[25] Sims, Christopher A. "The Precarious Fiscal Foundations of EMU, Working Paper (1999).

[26] Tanner, Evan and Issouf Samake. "Probabilistic Sustainabiity of Public Debt: A Vector Autoregression Approach for Braxil, Mexico, and Turkey." *International Monetary Fund Staff Papers* 55(1) (2008), 149-182.

[27] Woodford, Michael, "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy," *Economic Theory*, 43(3), (1994), 345-380.

[28] Woodford, Michael, "Control of the Public Debt: A Requirement for Price Stability?" in *The Debt Burden and Monetary Policy*, edited by G. Calvo and M. King, London: Macmillan, (1997).

[29] Yue, Vivian Z., "Sovereign Default and Debt Renegotiation." *Journal of International Economics* 80(2), (March 2010), 176-187.